Philosophy Dissertation

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On Incompleteness, Inconsistency and Moral Dilemmas.

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I

Moral Dilemmas are Inescapable

In this work I shall be examining the nature of moral dilemma. First, I will be tackling the traditional conception that moral dilemmas are symptomatic of an inconsistency of a moral code. Inconsistency is certainly a cause of moral dilemmas, but we shall see that consistency cannot fully protect against them. There is another root of dilemmas in incompleteness. After we are aware of this concern it soon becomes clear that to avoid moral dilemmas we want consistent and complete moral codes.

At this point I will turn to the work of Kurt Gödel and in particular his incompleteness theorems. These show that our desire to rid systems of both incompleteness and inconsistency is futile under certain conditions.

These conditions are, loosely, that a system be powerful enough to be able to justify itself and to be complex enough to deal with the infinite possibilities of the real world. These two conditions are almost exactly the requirements of a moral theory.

Finally I will turn to the consequences of such a discovery. Of these the most important is the fundamental dichotomy that has to exist in moral theory. A moral code can either be complete or consistent but not both. As it happens, although not by coincidence, this dichotomy is paralleled by the dichotomy between rationalist and intuitionist theories. Rationalist theories have ruled out inconsistency and intuitionists, incompleteness. The full effect of this has not fully filtered into the literature. Since at least one of these causes of moral dilemma is necessary, we are condemned to always be plagued by moral dilemmas.

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II

Difficult Decisions

There are many situations, some constructed, in which it is far from clear what one should do if one is to act morally. These are considered to be 'moral dilemmas.' There is a great body of work into the nature of these dilemmas but the main area of interest is whether there are any 'genuine' moral dilemmas. These are distinguished by contrast to apparent or prima facie moral dilemmas. In the general case a moral dilemma is a situation where a moral agent has obligations α and β but cannot fulfil both. The reasons for not being able to do both may be spatiotemporal – the agent cannot be in two places at once – or it may be moral – obligation β is the obligation not to do α. In either case, each of α and β considered on their own has to be possible and a moral obligation. Some examples will flesh out what is meant.

To highlight what is meant by spatiotemporal inability, consider Railton's 'Two Loans.'[1] Here he suggests that you have equal debts to two people, Pico and Young, which you obtained honestly with both the intent

[1] Railton (1996) pp 154

and the ability to repay. Between the loans being taken out and their repayment being due your expected income is greatly reduced. When it comes to repayment you have exactly half of what you owe; enough to pay exactly one of them. Here there are not two conflicting obligations, indeed there is only one – to pay of your debtors. It is a contingent fact that the same money cannot be given to two different people at once which means your obligations cannot be met.

To highlight a moral inability, consider Abraham. He is ordered by God to sacrifice his eldest son. His obligations are at odds here not because of some coincidental fact. He has a duty to his son, not to kill him, and he has a duty to God, to kill his son. It is the duties themselves contradicting which makes meeting all of them impossible.

Whilst we have two types of constraint they both share the property of not being instantly resolvable. The question remains as to whether they are resolvable at all. If they are resolvable after some rigorous contemplation then the dilemma was apparent. Again, this could take different forms depending on the moral theory but in any case one of the obligations could outweigh the other, one could not be a genuine obligation and so forth. If, however, after fully applying all the codes, laws or conventions to which your morality subscribes and after discovering all the facts relating to the situation, no correct answer reveals itself, you have a genuine dilemma. Buridan's ass stuck musing between two identical bales of hay can be thought of as such an example. However, this is only true in a particularly blunt application of choosing the best means of satisfying needs. Whilst there is no difference in the bales, or the effects of eating either, no convincing reason comes across to pick one over the other. The solution could come from the realisation that there is another more applicable principle by which to choose or that one of the bales was toxic. This is not a particularly moral decision, but the principle can easily be extended to Abraham. If there is a solution to Abraham's predicament it could either be derived from finding some new facts about the situation, like seeing the lamb God was providing as an alternative, or it could be derived from finding a relevant moral rule previously not presented, like his duty to his son actually being not to kill him unless God commands. In an ideal world you would come across the rule that said that in exactly this position, χ, one of the obligations does not apply in light of the other. In either case the situation merely appeared to be insoluble but in actual fact there is a unique correct course of action.

Genuine Dilemmas Arise From Inconsistency

There is another possibility. After contemplation, reasoning and fact finding there could still be no correct answer. Like, for example, if the bales of hay really are identical and there really are no other relevant considerations. In this case we have a genuine moral dilemma. Can these situations really exist? A large body think that they cannot – Kant, Mill, Aristotle and Aquinas are among them[2] – while an equally large body are convinced of the existence of genuine moral dilemmas – Marcus, Harman, Quinn, Donagan and van Frassen are among them. Loosely, rationalist theories that define 'good' as objective and claim that the best course is reached by reasoning tend to preclude the existence of dilemmas, although Rawls points out that voluntarist theories can fall into both camps.[3] Kant suggests that *"a conflict of duties is inconceivable.[4]"* Mill took it as fundamental to a moral theory that conflicts should not arise and built utilitarianism as a system to *"decide between…when their demands are incompatible.[5]"* There is a nuance in both these systems that needs to be considered as it has reverberated throughout the literature without due consideration. What Kant and Mill have subscribed to is the view that a moral system cannot give inconsistent instructions. That is to say that a

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[2] Gowans (1987) pp 5
[3] Gowans (1987) pp5
[4] Kant (1971) pp 39 of Gowans (1987)
[5] Mill (1961) pp 55 of Gowans (1987)

developed moral theory should not and, indeed, cannot have inconsistencies. The literature has fixed on trying to show that moral theories can be inconsistent. The one exception is Marcus who attempted to show that a consistent system can still leave us with dilemmas and we shall discuss her ideas later**.**

Formalising Moral Inconsistency

This inconsistency is traditionally formalised as follows.[6] '$O(\alpha)$' is the traditional way of indicating 'α ought to be done,' '$\mathcal{P}(\alpha)$' means α is permitted and '$\blacklozenge(\alpha)$' indicates that a is possible.

1)	Premise	$O(a)$	The requirements for a moral dilemma as characterised above.
2)	Premise	$O(b)$	
3)	Premise	$\sim\blacklozenge(a \wedge b)$	
4)	From 1 and 2	$O(a \wedge b)$	Agglomeration principle
5)	From 4	$\blacklozenge(a \wedge b)$	Voluntarist principle
6)	From 3 and 5	$\blacklozenge(a \wedge b) \wedge \sim\blacklozenge(a \wedge b)$	A contradiction.

[6] Brink in Mason (1996) pp 108, McConnell in Gowans (1987) pp 155 to 156 to name two.

There are two principles of deontic logic here which have been applied. They are the agglomeration principle in line 4 and the voluntarist principle in line 5. The agglomeration principle is that if you ought to do α and you ought to do β then you ought to do both α and β. This is formalised as:

$$\mathcal{O}(\alpha) \wedge \mathcal{O}(\beta) \rightarrow \mathcal{O}(\alpha \wedge \beta)$$

The voluntarist principle is the principle that 'ought' implies 'can.' In other words you can only be obliged to do things that are in fact possible. It formalises as:

$$\mathcal{O}(\alpha) \rightarrow \blacklozenge(\alpha)$$

Both of these principles seem intuitively plausible. Certainly an obligation seems meaningless if it is impossible to fulfil. The agglomeration principle, however, does not convince everyone. Brink draws a contrast between having obligations towards each of α and β rather than towards both.[7] Brink goes no further with his analysis. As such, there is still a strong intuition that the move to both is valid. McConnell suggests that this intuition is helped by the similarities between deontic operators, '$\mathcal{O}$' and '$\mathcal{P}$',

[7] Brink in Mason (1996) pp 109

and modal operators, '$\Box$' and '$\Diamond$'.[8] Some properties do hold in both modal and deontic logic:

	Modal	Deontic
(1)	$(\Box\mathcal{A} \vee \Box\mathcal{B}) \rightarrow \Box(\mathcal{A} \vee \mathcal{B})$	$(O(\alpha) \vee O(\beta)) \rightarrow O(\alpha \vee \beta)$
(2)	$\Box(\mathcal{A} \rightarrow \mathcal{B}) \rightarrow (\Box\mathcal{A} \rightarrow \Box\mathcal{B})$	$O(\alpha \rightarrow \beta) \rightarrow (O(\alpha) \rightarrow O(\beta))$
(3)	$\Diamond(\mathcal{A} \vee \mathcal{B}) \equiv (\Diamond\mathcal{A} \vee \Diamond\mathcal{B})$	$\mathcal{P}(\alpha \vee \beta) \equiv (\mathcal{P}(\alpha) \vee \mathcal{P}(\beta))$
(4)	$\Diamond(\mathcal{A} \wedge \mathcal{B}) \rightarrow (\Diamond\mathcal{A} \wedge \Diamond\mathcal{B})$	$\mathcal{P}(\alpha \wedge \beta) \rightarrow (\mathcal{P}(\alpha) \wedge \mathcal{P}(\beta))$

The key word is 'some.' There are properties that do not hold. The converses of (1) and (4) are cited by McConnell. At best we can formally show that the agglomeration principle is plausible, yet we cannot formally show it to be false either. It is because of this unclarity in the operators that means this formalisation does not end discussion about moral dilemmas. Brink has two other formalisations that shed light onto the issue.

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[8] McConnell in Gowans (1987) pp 158

The first:[9]

1)	Premise	$O(a)$	The requirements for a moral dilemma as characterised above.
2)	Premise	$O(b)$	
3)	Premise	$\sim\blacklozenge(a \wedge b)$	
4)	From 3	$b \rightarrow \sim a$	
5)	From 1 and 4	$O(\sim b)$	Obligation execution
6)	From 5	$\sim O(b)$	Weak Obligation
7)	From 2 and 5	$O(b) \wedge \sim O(b)$	Which is a contradiction

The second:[10]

1)	Premise	$O(a)$	The requirements for a moral dilemma as characterised above.
2)	Premise	$O(b)$	
3)	Premise	$\sim\blacklozenge(a \wedge b)$	
4)	From 3	$b \rightarrow \sim a$	
5)	From 1 and 4	$O(\sim b)$	Obligation execution
6)	From 5	$\sim P(b)$	Correlativity
7)	From 6	$\sim O(b)$	Weak permissibility
8)	From 7 and 2	$O(b) \vee \sim O(b)$	Which is a contradiction

[9] Brink in Mason (1996) pp 112
[10] Brink in Mason (1996) pp 113

In these two formalisations we have four new terms. They formalise thus:

Obligation Execution	$(O(\alpha) \vee (\beta \rightarrow \sim\alpha)) \rightarrow O(\sim\beta)$
Weak Obligation	$O(\sim\beta) \rightarrow \sim O(\beta)$
Correlativity	$O(\sim\beta) \equiv \sim\mathcal{P}(\beta)$
Weak Permissibility	$\sim\mathcal{P}(\beta) \rightarrow \sim O(\beta)$

The obligation execution principle essentially says that you cannot sabotage yourself. If you are obligated to do one thing then you are obligated not to do anything else that would prevent your doing it. Weak obligation states that if obligated to not do something then you are not obligated to do it. Correlativity is the assertion that obligation is identical with the impermissibility of not doing. Weak impermissibility states that if something is impermissible it is not obligatory. All bar the first are self evident. Obligation execution, to my eyes, is intuitively true but it is of an even weaker kind than the agglomeration principle. They both have assumptions about the meaning of the '*O*' operator.

If we are to use formal logic to establish the inconsistencies we have to properly define the terms we are using within the logic and then only use the rules of the logic to continue the work. The definition of obligation may actually not be the same for moral theorists, there is no necessity for us all to have exactly the same concept in mind when we use the word so long as in most cases our interpretations are similar enough. It is, however, the realm of the unusual case that gives rise to moral dilemmas and at this point our common conception of obligation no longer seems as certain as it once did. If our formal definition of *O(a)* does entail the agglomeration principle then it seems that the supposition of genuine moral dilemmas is inconsistent, but we could equally formalise our definition without such an entailment. What could a consistent moral system achieve?

Consistent Moral Theories

It seems that a plausible way to rid a moral system of dilemmas is to ensure that they are built without inconsistencies. So we should examine what a consistent moral system entails. Marcus argues that consistent moral theories do *"not entail that moral dilemmas are resolvable.*[11]*"* She puts forward that a consistent set of rules is a set of rules where there is a *"possible world in which they are all obeyable in all circumstances in that world.*[12]*"* She uses a trivial card game to drive the point.[13] In this game the

[11] Marcus (1980) pp 188 in Gowans (1987)
[12] Marcus (1980) pp 194 in Gowans (1987)
[13] Marcus (1980) pp 195 in Gowans (1987)

rules for trick winning are inconsistent. They demand that black cards beat red cards and that high cards beat low cards. There are three possibilities during each trick. Each of the rules could apply individually, they could both apply or neither could apply. If only one of the rules applies the game proceeds without a problem. If neither of the rules applies we have indifference and if both the rules apply there is conflict. Marcus then asks us what we would think of a contrived game which is still inconsistent but of more complexity such that the likelihood of the particular combination of cards which highlights the inconsistency actually occurring is very small. Here we have to clarify the metaphor. A single shuffled deck of cards represents a world. All the different ways that the cards can be shuffled is the set of all the possible worlds. So if there exists at least one way of shuffling, or stacking, the cards for which the conflict will not play out, the rules are consistent, by her definition. If a moral code, as she puts it,[14] is consistent in these terms contradictions and inadequacies may still arise for us. If the possible world in which conflict of the rules does not arise is not our world then the conflicts can and will occur for us.

This is a very subtle this argument. Marcus is tackling a different issue from the one she claims. Consistency is an important tool in her argument but the argument is more telling in its relationship to model

[14] Marcus (1980) pp 190 in Gowans (1987)

interpretation. A model is a way of thinking about a set of propositions in a tangible way. Here the propositions are interpreted as the rules of a card game and the model is the set of cards. In moral theory the propositions are interpreted as moral obligations and the model is, ideally, our world. One definition of consistency is that there exists a model for the set under at least one interpretation. In mathematics the consistency of a set of propositions is sufficient as the propositions are taken to define an abstract model. A set of propositions designed to be interpreted as a moral code has a far greater task. It cannot define an abstract model; it has to be modelled on the world that it deals with, ours. There is hope though. Whilst mathematics can define any consistent model without regard to the world, it is amazing how many parts of it do line up in some real tangible way with our actual world. Bridges can be built, planes fly, taxes get returned and so on. Whilst there is no explicit method for doing it, good principles or axioms exist.

Marcus carries on with her argument and highlights that in the game we can actually stack the deck, in order to stave off dilemmas; but in the real world we do not have that power. In a world this complex conflicts will arise. Here Marcus and I diverge. When a conflict arises in a game the players can step out of the game and make a new rule, or start again or simply stop playing. Marcus suggests that this is not possible in real life. While I agree that we cannot start again with a different deck or leave the table, we can invent a new rule. Consider Euclid's five axioms. These axioms are consistent with each other but the model they imply is not universal. In particular, as we live on a globe, parallel lines do cross. Consequently there are other geometries which themselves are consistent for which a sphere is a model. Geometries are invented but with strict indications of where they apply. We have not abandoned Euclid's original geometry as it is still the perfect system for flat planes. Similarly we can conceive that it would be possible for a hitherto perfect set of rules to be unable to fit some newly discovered feature of the world. Imagine a new ability, like genetic engineering. Let us assume that the original rules did not have a way of dealing with such a concept, but this was never noticed as the problem never arose, it could not have arisen. So, as in geometry, we can conceive that either a slight change to the existing rules or a new set of rules which now include solutions for genetic engineering. We do not abandon the old rules; they have, after all, served us so well; we have just become aware of where they are no longer effective.

There is, however, one caveat. With differing geometries there are clearly defined spaces to which they apply. The models are known. They are all together part of mathematics, itself an axiomatic consistent[15] system. The whole system includes the rules about where the geometries apply, so we

[15] This is not as clear as I have suggested here. A full discussion on the consistency of logic is included further on.

actually have one coherent system, not different ones for different fields. As it is for mathematics it is for morality. We do not have two different moral codes, one for each occasion. Instead we have one larger moral code including when rules about when the new additional rules apply. If we consider this set perfect is there any reason that another unforeseen dilemma will not arise? Of course, there is not. It may seem that the problem arose here because the consistent code did not model our world but some other possible world. In fact even if our world is the simplest and best fitting model we cannot get round the problem of an unforeseen dilemma.

In short, consistency is necessary but not sufficient to guarantee that there cannot be moral dilemmas.

Genuine Dilemmas Arise From Incompleteness

Earlier we saw that inconsistency in a moral system can lead to moral dilemmas. We also saw that many thinkers insist that morality must be consistent. Finally we showed that even a consistent moral theory will not prevent moral dilemmas occurring. What other form can these dilemmas take? Incompleteness, a property closely related to inconsistency, is the other major cause of dilemma.

So far we considered the notion of conflicting obligations. However, our obligations do not have to conflict each other if they are to leave us without moral guidance. Consider again Buridan's ass. If we strictly interpret the obligation to 'always choose the best act,' we are not actually presented with a contradiction. The ass is not obliged both to eat one bale and to eat the other. What is actually returned is no obligation. The rule simply does not give an answer to the question, 'which bale should the ass eat?' It is like Marcus' card game when two red deuces are played. The rules simply do not state which one is the winner. This is the notion of incompleteness. That there are questions about which the rules cannot give an answer even though the question appears to be of the sort it should. Again, we have to make the transition from the not particularly moral to the hard stuff.

Consider Kant's categorical imperative, particularly, for the sake of example, the humanity formulation. We are to treat humans as ends in themselves and never (merely) as means. How would a moral system built around this help us decide whether to drive or take the bus to work? What about whether to eat farmed or wild fish? We shall have to assume here that all the staff involved are voluntarily employed; are paid a fair wage; love their work and so on. The categorical imperative, so construed, does not give an answer. It does not give conflicting answers – by no means are we compelled both to eat and not eat farmed fish. Here the incompleteness may seem benign. We are simply permitted to do either and there are no moral implications. Yet we can easily scale up the problem of not having an answer if we look at utilitarianism.

Utilitarianism can never give inconsistent obligations. In any situation there cannot be two options that are better than each other. At most one from any pair of options can be better than the other. However, what happens if the two best options are equally good – or have an equal score in the utility calculus? Again, we have no clear answer and, again, we do not have conflicting obligations to both do something and to not do it; we simply have no obligation towards either. As utilitarianism has no fundamentally wrong types of deed – at least not in its purest simplest sense – we can consider situations where huge numbers of lives are on the line. If it were not equal there could easily be an obligation to kill millions of people to save the lives of billions without the qualms of Kant having not to harm innocents. It seems that a situation can be constructed with many lives on the line and the best advice we can get is that it does not matter. Surely we would expect a rigorous moral system to be of help in what must be a very weighty moral issue? So is it possible to construct a moral system that is immune to these two causes of moral dilemma? It turns out that there are features of rational moral systems that preclude this. These are what follow.

Rationalist Moral Systems Are Axiomatic

All moral systems share in common some key features. One we need to look into is that they cannot contain comprehensive lists of every possible situation that will arise. That is to say all moral theories contain guides that are, or are almost, universal which are to be applied and reapplied. For Kant it is the categorical imperatives, for Mill it is utility and for Aristotle it was the virtues. It is not without good reason that moral theories have this feature. If it were possible for a comprehensive list to be made it would be impossible for any moral agent to remember all of them and equally impossible to fit them all in a convenient book. Every possible situation includes a lot of situations that people will never experience but even a comprehensive list of likely situations is impractical. Instead of such unwieldy notions we extrapolate from general rules to specific instances. If we consider one system we can see how this works.

Utilitarianism is a consequentialist moral theory that has its only guiding principle that one should cause as much good as possible. A fully formed utilitarian system will have a definite definition of good and how to measure it. We will here stick to Mill's own choice of utility, as it is not our purpose here to refine a moral system. The fully formed definition will have a defined method of calculating utility. It will also have a formalised decision rule. Usually this is along the lines of 'from all the available courses of action do the one that leads to the most utility.' This is all that is required to define accurately a utilitarian moral system. It feels more comfortable to do this to utilitarianism than other systems as the utility calculus already sounds like mathematics, but in truth all rationalist moral systems can be characterised this way.

A moral system is fully developed when it is able to help guide people. If it is to be a useful guide it needs to have a decision system. Rationalist, realist theories are by their very nature systematic and, therefore, logical. This need not mean cold; there is no necessary constraint on the subtlety of the logic. It can include various aspects of emotions; guilt, blame, hurt and so forth. Indeed, moral systems must be extremely complex. All that I am asserting here is that a developed moral theory cannot break fundamental logical entailments. So, if moral systems are complex logical systems, might their consistency depend upon the consistency of the logical systems? The consistency – and the closely related concept of completeness – of logical systems were a major field of research in the late 19th and early 20th century.

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III

Consistency and Completeness of Logical Systems

Logical systems are a set of axioms and a set of rules governing the interaction of these rules. In theory, the axioms can be any logical statement but in useful systems, like Euclidean geometry, they correspond to some basic truth. The choice of these axioms is important to the success of the system. If they cannot be interpreted as reflecting anything in the real world then the system is of limited use. From a set of axioms and the interaction rules further statements can be derived. When you come across a new statement you want to be sure that it is consistent with other existing statements. It is also useful to be able to find out if a statement that looks like it may be part of the system actually is. This can be tested with the axioms. Of the complex statements some may be provably part of the system and others may be provably not. There may be some statements which can not be shown to be part of the system but can neither be shown to be inconsistent with it. In the early twentieth century it was to this class of statements that mathematicians and logicians turned.

The desire was to show whether mathematics was complete and consistent. If a system is complete then every statement which is part of the system is provably so. A consistent system is system that is unable to derive contradictory statements. By the middle of the 1930s Kurt Gödel had proved that these wishes could never be accomplished.

Formalisation

Early attempts at proving the consistency of axioms were made by Hilbert.[16] If one models a set of axioms, that is to contrive a 'real' object or set of objects which satisfies the axioms, and inspects it one can exhaustively show that all members are consistent with each other. This method only works for very small systems. There need to be few enough members that all can be inspected thoroughly. Establishing the class of all prime numbers less than ten to be a set of four members is possible as there are only ten candidates to inspect. More complex sets cannot be modelled in this way. The relatively simple postulate: 'every integer has an immediate successor which is different from every previous integer,' cannot be shown to be shown to be consistent by inspection or modelling.

[16] Nagel and Newman (1958)

Hilbert formed another way of proving consistency. You can translate the five Euclidean axioms into Cartesian algebra. This is achieved by redefining ‘point’ to mean ‘pair of numbers’ and ‘line’ to mean ‘the linear relation between two numbers expressed by a first degree equation with two variables.” Whilst this does not seem to be any simpler while we are using words, in symbolic representation Cartesian algebra can be very easily manipulated and new postulates can be translated into it. It is like a universal language. Any new postulate can be manipulated to show that it is consistent with the initial five.

What, then, had Hilbert shown? At this point we are able to show, in theory, whether or not any new postulate is consistent with the five axioms. Therefore any consistency proof is a ‘relative’ consistency proof – that is, a postulate is consistent relative to some other axioms. As yet there is not a proof for the consistency of those axioms. Those axioms are still the five Euclidian ones. These have been accepted as true because they appear to self evidently fit the model of the real world. However, given the size of the real world and the limited experience people have had with it, we have not fully inspected the model and cannot be sure that the real world is a perfect model. As we are unsure of the match between the model and the axioms we cannot be sure that the axioms do not derive inconsistent postulates. What is required, therefore, is a tool that can deliver absolute proofs of consistency.

As models are the cause of the relative nature of the earlier proofs Hilbert eliminated them completely by a process he called "*complete formalization.*[17]" To do this one takes all meaning out of the symbols. All axioms are expressed as strings of symbols and are to be treated as nothing more. The rules of the system are to be expressed as ways the symbols are allowed to be arranged or how strings may be modified. This is all expressed in terms of the layout of the symbols graphically and without reference to what they mean. The rules about a formal system need not be part of the system and it is considerably easier if they are not. For example the string:

$$x = x$$

belongs to formal arithmetic but the statement:

The sign '=' must have numerical expressions on either side.

is a statement about arithmetic. These statements 'about' something rather than being part of something Hilbert prefixed with 'meta.'[18] The statement above was meta-arithmetical as it was about arithmetic. From this, it should be clear to see that the statement *Arithmetic is consistent* is not an

[17] Nagel & Newman (1958) pp 26
[18] Nagel & Newman (1958) pp 28

arithmetic statement. In turn it shows that arithmetic can not show itself to be consistent but some meta-arithmetical reasoning is required. Hilbert's plan was to show within mathematics all the structural relationships of the axioms, which could be expressed as a *"geometrical"*[19] pattern of formulas. This pattern would be expressible within arithmetic and its consistency could be tested.

Codifying

Now it is clear that a systematic formalisation of a system is required to test said system for consistency. We must carry out such a formalisation. Frege, Boole, Russell and Whitehead were already working on this for other motives. We can consider a well known simple proof to show these reasons.

[19] Nagel & Newman (1958) pp 33

Euclid's proof that there is no greatest prime number goes as follows:

1. Suppose χ is the greatest prime number.
2. Multiply together all the prime numbers up to and including χ and add *1* to the product, call this y.
3. If y is prime, it is a greater prime number than χ.
4. If y is not prime, it must have prime factors. At least one of these, z, is not a factor of y-*1*. z must be a bigger prime than χ.
5. y must be prime or not prime.
6. Hence, χ is not the greatest prime.
7. There is not a greatest prime.

Whilst all the rules governing the deduction were obvious to Euclid, and indeed most lay people, they were not codified. Consider line 5. This is a logical theorem and a rule first formalised by Boole in 1847.[20] The theorem is 'P $\vee$ ~P' and the rule governs substitution of variables. The whole proof is in fact a substitution of a logical theorem $(p \rightarrow r) \rightarrow [(q \rightarrow r) \rightarrow ((p \vee q) \rightarrow r)]$.[21] This kind of analysis was only available post Boole.

[20] Nagel & Newham (1958) pp 40
[21] Nagel & Newham (1958) pp 104

Frege set out to show that all arithmetic can be described in terms of Boolean logic. This was something that was of no import until the need for a consistency proof appeared. Logic is a particularly awkward way of expressing mathematics. It does, however, have the advantage of being able to express mathematical argument as well as the mathematics creating the appeal for such a task.

An apparently simply notion of the cardinal ‘1’ is defined thus; the class of all classes similar to any unit class. Class, unit class and similar are all notions that were predefined in Boolean logic. Russell and Whitehead finally completed the task of reducing mathematics to general logic, and as such dependent on the same axioms, and published it in *Principia Mathematica.*

Absolute Consistency

What are ultimately required in a fully formalised system are three things. Firstly a vocabulary needs to be defined. This is an exhaustive list of all the symbols that will appear in the completed system. These come in three main types; variables, connectives, and punctuation. In mathematics χ, +, and (, are an example of each respectively. Secondly formation rules to indicate how a string is formed. These simply say which symbols may appear next to which other symbols. They do not make any claims about the validity of such strings. For example *2+2=4* and *2+2=5* are both well formed but one is not a formula in standard mathematics. *2+1+=3* is a poorly formed string. Thirdly and finally rules of transformation are required. These govern what changes can be made to a well formed string. In mathematics this can be seen as being the rules that allow $(a+b)^2=0$ to become $a^2+2ab+b^2=0$, although in this form mathematics is not a fully formalised system, as the symbols are interpreted. In sentential logic there are, in fact, only two transformation rules. One is substitution, which says that any well formed string can substitute for any variable as long as it is substituted uniformly for each occurrence of the variable. The second is Modus Ponens, which states that if we have two strings, $\mathbf{S_1}$ and $\mathbf{S_1 \rightarrow S_2}$ then we can derive $\mathbf{S_2}$.

So far this system does not produce anything, what is required is some basic formulas from which others can be derived using the transformation rules above. These are the axioms. The axioms are well formed strings and formulas, only strings which are derived from them are formulas.

In *Principia Mathematica* they include:[22]

1) $(p \vee p) \rightarrow p$

2) $p \rightarrow (p \vee p)$

3) $(p \vee q) \rightarrow (q \vee p)$

4) $(p \rightarrow q) \rightarrow ((r \vee p) \rightarrow (r \vee q)$

We now have the power to prove the consistency of logic. The formula $((p) \rightarrow (\sim p \rightarrow q))$ is a formula of sentential. From this if we assume that there is a contradiction, that is both p and $\sim p$ are formulas we can derive q from the transformation rule Modus Ponens. What this means is that if there is a contradiction then any well formed string can be derived from the axioms, and is therefore a formula. Therefore, in order to show that there are no inconsistencies we simply need to find a well formed string and demonstrate that it is not a formula. The simplest tests for being a formula are meta-mathematical. We need to observe the external properties of a formula which at least one well formed string cannot have. In order to know that all formulae have this property it must be present in all the axioms and

[22] Nagel & Newham (1958) pp 48 NB this is not all the axioms in *Principia* but a selection capable of expressing sentential logic but not arithmetic.

hereditary. All the axioms have fewer than five connectives, but this is not hereditary as substitution allows strings to grow indefinitely.

All the axioms are tautologous in the tight logical sense – they are true regardless of interpretation or substitution. Truth is not a formal notion; we have let an interpretation in. We are looking for absolute proofs and any proof requiring interpretation leads to the same problems Hilbert had. Interpreting a system simply moves the burden of proof to the interpretation. Tautology can also be defined in a purely systematic way. We can remove the notion of truth and all its connotations by defining two classes, K_1 and K_2. These classes are exhaustive and exclusive. There are three rules for determining membership:[23]

1) **~S** belongs to K_2 if **S** is in K_1 otherwise it belongs to K_1.
2) $\mathbf{S_1} \vee \mathbf{S_2}$ belongs to K_2 if both $\mathbf{S_1}$ and $\mathbf{S_2}$ belong to K_2 otherwise it belongs to K_1
3) $\mathbf{S_1} \rightarrow \mathbf{S_2}$ belongs to K_2 if $\mathbf{S_1}$ belongs to K_1 and $\mathbf{S_2}$ belongs to K_2 otherwise it belongs to K_1.

[23] Nagel & Newham (1958) pp 110 NB. For consistency the membership criterion here are only those that apply to the axioms in this text and therefore the rule governing the connective '·' is not included.

A tautology is defined as a string which is a member of K_1 independent of the class of its constituents. It should be clear that a tautologous nature is not lost when substituting. A little reasoning will show that tautology is also hereditary for modus ponens.

1) Suppose $\mathbf{S_1}$ and $\mathbf{S_1 \to S_2}$ are tautologies.
2) From the third rule of classes $\mathbf{S_1 \to S_2}$ and $\mathbf{S_1}$ can only both belong to K_1 if $\mathbf{S_2}$ belongs to K_1.
3) Therefore $\mathbf{S_2}$ must always belong to K_1 – i.e. is a tautology.

We now have that being a tautology is hereditary. It is fairly trivial to demonstrate that the axioms are all tautologous. Here is a table demonstrating that the first axiom is a tautology:

p	$p \vee p$	$(p \vee p) \to p$
K_1	K_1	K_1
K_2	K_2	K_1

The second column is produced by inputting the class of p into the second rule. The third column is derived using the third rule.

Having shown that all formulas must have the property of being a tautology we now need to find any well formed string without this property to show that these axioms are consistent. $p \vee q$ is an example. These four axioms are consistent with each other, absolutely. Let's turn our attention to a more complex system.

Gödel Numbering

In order for a system to prove its own completeness it needs to be self able to directly deal with the formulas that make it up. That is to say it needs to be self referential. The drive for completeness proofs and axiomatic method was to prove it for maths, starting with arithmetic. As arithmetic is the logic of integers the references need to be numbers. As the labels are numbers arithmetic analysis can be carried out upon them. The numbers have to have a one to one mapping onto each formula. It would be useless if a label referred to more than one formula and similarly if a formula has two labels then each must have different arithmetical properties, rendering it immune to analysis.

In order guarantee the uniqueness of each label, Gödel utilised the notion of prime factors. Each integer is the product of a unique set of prime factors. In order to produce unique numbers for a system we have to multiply together only primes. First the vocabulary needs to be numbered:

Constants	**Symbol Number**	**Interpretation**
~	*1*	Not
∨	*2*	Or
→	*3*	If…then…
∃	*4*	Existential qualifier
=	*5*	Equals
0	*6*	Zero
s	*7*	The successor of
(	*8*	Punctuation
)	*9*	Punctuation
,	*10*	Punctuation
Numerical Variables[24]	**-----**	**Possible Substitutions**
x	*11*	*0*
y	*13*	*s0*
z	*17*	*y*

[24] There can be infinitely many variables as long as during a body of work they are consistently labelled with consecutive primes above ten raised to the appropriate power.

Sentential Variables[24]	**-----**	**Possible Substitutions**
p	11^2	$0 = 0$
q	13^2	$(\exists x)(x=sy)$
r	17^2	$p \rightarrow q$
Predicate Variables[24]	**-----**	**Possible Substitutions**
$\mathcal{P}$	11^3	Prime
$\mathcal{Q}$	13^3	Composite
$\mathcal{R}$	17^3	Greater Than

These numbers are going to be used as powers to raise primes. For a string of length x we use the first x primes. We then raise the first prime to the power of the number that corresponds to the first symbol, the second prime the second symbol and so on. For the sentence $(\sim\exists x)(0=sx)$ this reads:

Symbol	(	~	$\exists$	x	)	(	0	=	s	x	)
Symbol Number	8	1	4	11	9	8	6	5	7	11	9
Prime	2	3	5	7	11	13	17	19	23	29	31

Gödel Number: $2^8 \times 3^1 \times 5^4 \times 7^{11} \times 11^9 \times 13^8 \times 17^6 \times 19^5 \times 23^7 \times 28^{11} \times 31^9$

From the numbering of each formula we can make two more advances. Any sequence of formulas – like a proof – can be similarly numbered and any relationships between formulas can be expressed purely mathematically. For sequences of χ formulas we again take the first χ primes but this time we raise them to the Gödel number of each whole formula. A proof is a sequence of formulas that leads from the axioms to the formula proved. As each transformation step is an arithmetical relation, albeit in most cases complex, there is an overall arithmetic property of proof sequences in general.

At this point it must also be noted that new symbols can be added to the system, but only in a strict manner. These symbols are abbreviations of long formulas. For example we can write the cardinals as *'1, 2, 3…'* instead of *'s0, ss0, sss0…'* These new symbols do not have separate Gödel numbers. The Gödel number for *'1'* is $2^7 \times 3^6$, as derived from *'s0'*. As long as the new symbols are used only as the formulas they abbreviate and not with all their common language connotations, one can reason using them. One such abbreviated symbol is $\chi \mathcal{B} z$. This translates as: χ is the Gödel number of the proof of the formula whose Gödel number z.[25] This formula is very functional in Gödel's proof.

[25] Gödel (1931) pp55

The Crux of Gödel's Proof

xBz can be generalised further if we use the Gödel number of *xBz*. To do this we replace *z* with its numeral. Gödel used the abbreviation *Gen* for this. *(11 Gen r)*[26] represents the Gödel number of the formula we get if in a formula all the occurrences of *x* (defined as *11* in our vocabulary) are replaced with *r*, where *r* is the Gödel number of the formula. The symbol *'r'* is chosen to stand for recursive. Now we need to put it all together.

First hypothesise that there is an unprovable formula. There would be no Gödel number that corresponds to a proof of the unprovable formula:

(~∃x) (∃z)(xBz).

If we make a stronger claim that a particular formula is unprovable we simply replace *z* with the numeral of that formula. If we input '*(11 Gen r)'* as the numeral we get:

(~∃x)(xB(11 Gen r))

[26] Gödel (1931) pp51 NB In Gödel's proof 17 is used in place of 11 as his vocabulary was different. He made do without '∨', '=' and '→'and only odd numbers were used as labels.

This statement reads in English as: there is no number that corresponds to the Gödel proof number of this statement. If we examine what this means for the system we have an astounding result.

If the statement is true, then it is unprovable as that is its claim. This means that there is an unprovable true statement. The system must be incomplete. If the statement is false, then a proof exists. This would mean that there exists a provably true false statement. The system must then be inconsistent. The consequence of this is that any axiomatic system powerful enough to include statements about its own validity must either be incomplete or inconsistent.

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IV

The Future of Moral Systems

What are the implications of this result for moral systems? If we accept that moral systems cannot be at odds with the fundamental laws of logic, we have to accept that a sufficiently powerful moral system is necessarily incomplete or else it is inconsistent. To what moral codes can this apply? I would suggest that it exists as a limit to all of them. First consider an insufficiently powerful enough moral system. That is to say one simple enough to be both complete and consistent. Such a system will be deficient.

The Moral Sphere

If we rephrase the definition of completeness for system we can see the first deficiency. A formally complete system will have no undecidable propositions. I wish to make explicit one thing that this does not say. It does not say that there are no propositions that are not part of the system. For a trivial example *2+2=4* is part of arithmetic but *"Grass is green"* is not. For any complete system there are still going to be statements it will not hold answers for. This in itself is not an issue. A complete system designed to efficiently file documents is not deficient if it does not give indications about when to plant crops. Similarly a moral system need not be deficient if it does not give answers to non moral questions, as long as it can give answers to all moral questions. We do, however, come across issues that make this impossible. The problem becomes what counts as a moral decision – or even broader what is a moral question?

There are clearly statements on either side of the divide. "*Abortion is always wrong*" is clearly a moral statement where as "*Marblehead is north of Boston*" clearly is not.[27] There are different ways to define what will count as a moral statement. These include things like: [28]

1) 'moral statements must include a value term;'
2) 'moral statements must be about some moral issue;'
3) 'moral statements must be about a moral custom for society and contain a value term;'
4) 'moral statements must contain an imperative;'
5) 'moral statements must contain a categorical imperative.'

[27] Both taken from Feldman (1978) pp 2
[28] Adaptations from Feldman (1978) pps 2 to 8

All of these possibilities are far from ambiguous and some are clearly insufficient definitions. They have terms that need to be defined, ‘value term,’ ‘moral issue,’ and ‘categorical imperative.’ Some definitions of morality have a smaller scope than others. Kant’s humanity formulation of the imperative, for example, only deals with issues involving humans. This limits the moral sphere to human concerns. There are, however, large numbers of people, including serious philosophers, who insist that animals have rights – there is even a school of thought that professes that inorganic landscapes have rights. If there were a self evident limit of the moral sphere, or even one requiring a convoluted proof, we could not continue these debates.

There are two ways out of this particular problem. One is to define morality as that which is delineated by the complete moral system and the other is to insist that a moral system does include everything. Unfortunately neither of these routes fully rids us of the problem.

The Moral Infinite

Trying to expand to the infinite can only be done at the cost of completeness, which is exactly what we were trying to avoid. If we consider a strong act utilitarian moral system we remember one of the criticisms is that it is too demanding. Every decision about everything has some value in the moral calculus – the moral system does include everything; our second solution. Yet, we have already seen that utilitarianism leads to undecidable propositions. It is not just utilitarianism that leads to the undecidable. If a moral system is to be infinite it must surely include the moral grounds for constructing moral systems, accepting one and living by one. Unfortunately this kind of reasoning has exactly the recursive nature that makes a system vulnerable to Gödel's theorems. If the expanding to the infinite undermines the possibility of completeness perhaps the other option will help.

Defining morality to be only that which is delineated by a complete system may resolve our issue, but only technically. We still have the intuition that morality is not the same as mathematics. It appears that we may be able to conceive of a limited system that is complete and consistent but we do not seem compelled to think that it can capture all of morality. We constantly express this when we find examples that we insist are moral questions but are beyond the scope of the moral theory we are attacking. Beyond our simple unwillingness to accept a limited moral sphere there are further reasons for doubting one. If, from our hypothesis, the system is complete it cannot be both reflexive and contain the power to justify propositions. A moral theory, we would all admit needs the power to justify things – otherwise how else are we do discover the 'right' or the 'good?'

Therefore a complete moral system must not be reflexive. How is such a system justified? From the constraints of consistency a moral system is not able to prove itself. This means that there must be a higher reason, a higher justification for things. The nature of a more important value or reasoning than morality sounds absurd but that is not reason enough to dismiss it. However, if there actually were a higher standard than morality that could prove morality would this not be in interesting thing to study? If we were to study it we would surely come to the same questions. We would want to know if this higher system was consistent and complete. There would have to be an even higher power to justify that power. This can only continue. Indeed this was why modelling Euclidean Geometry in Cartesian Algebra did not solve the problem for Hilbert.

It seems to be inescapable. We are condemned to either an incomplete or inconsistent moral system.

Incompleteness in Action

It may seem here to be all doom and gloom once we allow incompleteness into moral theory. This need not be the case. It certainly would have been desirable to have avoided it but we can incorporate incompleteness into the moral world. Indeed, in many ways, we have always had clues that it was there.

Once we are stuck with incompleteness we have to accept the consequences. Let us jump straight in with the situations we were originally looking to avoid; situations where no amount of reasoning will give a clear answer. We shall again start with a trivial situation. Blackburn gives us two examples useful for this discussion.[29] The first is picking between two tins of baked beans in a supermarket. The situation is drawn up to be symmetrical. The beans are in fact from the same factory, produced the same way. In every regard the beans inside each tin are identical. There is no reason to choose one over the other, yet we readily pick one and move on with our lives. This is called a *"stable objective quandary.[30]"* The lack of a good reason does not bother us as we believe the situation to be stable. This notion starts to get more interesting if we are choosing between buying one of two new cars. We, after much consideration, can find little difference between the two – at least we cannot discern which one is going to be better. Eventually, of course, our old car will eventually fall apart and we have to pick one of the new cars. Regardless which one we choose we will always have a feeling that we should have bought the other. We experience regret. Notice that this feeling does not depend on which one of the cars was

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[29] Blackburn in Mason (1996) pp 127 - 128
[30] Blackburn in Mason (1996) 128

actually better or even if the cars were equally good. What these decisions leave is some form of residue.

Now, as is required, we must move up to clearly moral decisions. Agamemnon and Sophie both had difficult decisions to make. In either situation no matter which decision they will have made they will experience regret about what they had to do and guilt about doing what feels like a wrong thing. In the first case, let us assume that there was a correct course of action and it was not taken. Here the feelings are easy to explain by their association with moral misdeeds. However, if they made the right choice what are they actually regretting? It appears that they regret doing the right thing; they feel guilty about making a strong good decision. Surely this is a case for pride? It is even more perverse to think about the regret when there was not a right or wrong choice. Of course, guilt is a misleading word chosen to highlight the point. There is the emotive sense of guilt and there is the judgemental sense of guilt. The judgemental sense is the declaration that someone has actually done something wrong. The similar emotion often comes attached with the same connotations. The attachment is so strong that the feeling guilt has been used in the case of moral dilemmas points to there being a correct answer. The guilt is a sign that we really did do something wrong. However, being racked with guilt is not the same thing as being guilty of wrongdoing. It seems strange, though, to use our irrational unreasoned emotions to point to the existence of a rational answer. To me the fact that we experience guilt and regret when in situations when we have done no wrong – or even done great good – is a symptom of the incompleteness of rational moral systems and the inconsistency or emotional and intuitive moral systems. This is not a judgement that one type is better than the other they are just differently equipped. Remember Gödel's proof just says that a system has to be either incomplete or inconsistent, it does not say which is type of system is more suitable for what purpose.

It is not just the irrational emotions that cause the dissimilarity between the right decision and guilt. We can rationally be guilty even when we have done no wrong. This example adapted from Blackburn[31] highlights the point. Two friends, offer to lend me something. In all respects the offers are identical. I only need one so I only take one for what could be a good reason outside of the offer. If one of my friends lives on my route to work, it is easier for me to accept their offer than the one from the friend who does not. Even though I have clearly made the right choice the friend whose offer I did not take up will feel spurned and I will have to make some token of apology. Here my apology expresses some guilt, however slight, I have for not taking up the offer. The fact that I did nothing wrong in not taking the

[31] Blackburn in Mason (1996) pp 131

offer does not stop this. This guilt is also not of the emotive irrational type, it is considered and reasoned.

Guilt, Blame and Punishment

If rational guilt and therefore being guilty is not directly connected with wrong doing how are we to fit guilt into part of the moral system? This carries equal weight for the related notions of judgement, blame, praise and punishment. What we have to notice here is that in an unbound system decisions to judge, blame and so on are decisions which can and perhaps need to be decided within the moral system. Some systems suggest the people lose some of their rights when they do wrong. Contract theorists like Hobbes and Rousseau will have it that by not following certain obligations you forfeit the reciprocal obligations. For Kant punishment is only related to wrongdoing.[32] Any other reasoning is using the offender as a means. Yet neither of these characterisations helps with the example. I did not fail to fulfil an obligation – my side of the contract – and my apology seems to be using me as a means to placate my spurned friend. Utilitarianism is, as always, superficially much simpler. Remember that in utilitarianism all decisions are moral. So my decision to blame someone, or the state's to punish, is not derived from them doing wrong. What matters is whether or

[32] Rauscher (2007) § 7

not punishing, blaming or ascribing guilt to someone brings about more or less good in the calculus. To highlight an extreme example consider having had the opportunity to kill Hitler as a baby. Making some assumptions that no other even more evil dictator would come to power instead you could save the lives of many Europeans who died either from the war or the holocaust. It seems clear that that you should take the opportunity. However, it seems very plausible that a utilitarian state should also punish you for the murder, as the effect of not doing so may be the appearance that infanticide is condoned.

In both Kantian and utilitarian moral systems punishment has to be morally justified. There is no necessity that the punished has to have acted against the moral system. Kant happens to have thought that this was a condition otherwise one would undermine the humanity of the guilty, whereas utilitarian thinking does not make the link between guilt and wrongdoing. This all shows that our emotional responses to moral decisions go beyond the limits of rational moral theory.

The Future of Moral Systems

There are two courses of action available to us once we have acknowledged the importance of Gödel’s theorems. Any moral system we develop must be either incomplete or inconsistent. Both options are available to us. Intuitionist and voluntarist theories that have always embraced traditional moral dilemmas have done so because they consent, however tacitly, to some form of inconsistency, although Rawls made an attempt at a consistent rationally constrained voluntarist system.[33] Rationalist theories, which have denied traditional dilemmas, have dismissed inconsistency and must embrace some form of incompleteness.

This dichotomy is not unparalleled. In the two extremes mathematics and logic have 'opted' for an entirely consistent system, and therefore have had to accept the existence of unprovable conjectures. Natural languages, however, have, out of the need to express everything, ended up inconsistent. When reading about bitter sweet sensations or objects falling from the floor up to the ceiling our brains do not go into melt down over the apparent contradictions. There are, of course, costs of this. We cannot ever be precisely sure that two people are talking about the same thing, a fact the television programme QI generates most of its questions from. "Name a berry," they say knowing full well that the contestants will jump onto things like strawberries and raspberries, which by some particular definition are not berries.

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[33] Gowans (1984) pp 5

What are the relative costs and benefits of either incompleteness or inconsistency in a moral system? The benefit of an inconsistent system is that there are no unanswerable questions, or there need not be – a poorly conceived system could be inconsistent and leave out a great many of important issues. The intuitive, emotive response will always exist. Part of the appeal of intuitionist theories is that they often suggest that they are natural. One can cite evolutionary demands which have honed our emotions to be far more subtle than brute logic. One can suggest that the power of the human mind is the heuristic instant responses it makes to varying situations, resorting to logic is lowering our status as sentient beings to those of automatons. Whatever justification one gives for a complete intuitionist or voluntarist moral theory one cannot escape the fact that our intuitions are bound not to be consistent. There is no telling that the next case will not bring out a response at odds to one had previously.

Incomplete consistent systems will never have the traditional moral dilemma problem. There can never be, as Kant foresaw, genuine conflicting moral obligations. However, this too comes at a cost. There is the necessity of undecidable maxims. From any set of basic principles one will want to abbreviate them. Claiming that ‘do not lie’ is a rule is really a conjecture that from the fundamental autonomy of will and respect for humanity not lying follows – not lying is entailed, on other words, from the fundamentals.

Some of these conjectures are, in virtue of incompleteness, unprovable. It is not that they are neither right nor wrong it is just that there is no way of discovering which. This limitation condemns us to the existence of a form of moral dilemma where we do not have contradictory obligations we simply do not have obligations at all. Just like voluntarists and intuitionists have to accept that someone looking at their actions logically will point to contradictions, rationalists will have to acknowledge that they are going to have irrational emotional attitudes towards moral situations. These need not be part of the decision making process but their existence cannot be ignored if the system is to be fruitful.

We should also note that there is a little bit of overlap between in the dichotomy. In a strict consequentialist system the negative consequences of doing a thorough calculation at every decision have to be taken into account. The time spent considering which course of action to take could have been spent doing even more good or during that time the facts may change making the result no longer relevant. The act of adopting a quick method of evaluating the situation is no doubt a beneficial one. If someone or some group were to do such a calculation it is perfectly conceivable that the result would be to adopt intuitionism in some form. If over the course of a lifetime the benefits of following your instinct outweighed the costs of the occasions when it delivered a different decision than a specific calculation would have done, the act of adopting intuitionism is a good thing to do. Here we would have allowed inconsistency into an already incomplete system so both forms of the dilemma arise, although in a practical sense one will only have problems with inconsistency, as in cases symptomatic of incompleteness one's instinct will be followed.

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V

Coda

Difficult decisions are never going to go away. We are always going to have to face them and hope we have the moral courage to tackle them. The examples may seem tired but they will never completely disappear. New ones will be used as moral theories become more subtle, or as the focus shifts, and they will never lose their poignancy. We have seen that the classical assumption that moral dilemmas arise from inconsistency does not address the entire problem. Certainly, inconsistent obligations would give rise to insoluble dilemmas but eradicating inconsistency is not enough. Even consistent systems lead to insoluble moral dilemmas. Gödel's theorems have shown that consistency is not compatible with completeness.

It appears that this knowledge has always been there, with intuitionists talking past rationalists and vice versa. The rationalist points to contradictions with what an intuitionist does and the intuitionist points to questions that the rationalist cannot breach appropriately. Perhaps part of the problem stems from what we want a moral code to do. Yet however much we may know the impossibility, we continue to want a clear unique answer in every situation we encounter.

We can also see that systems of justice must fall foul of the same concerns as they have to consider the morality of both the agent accused but also of the judgement made. They, perhaps more clearly, are defeated by their own recursive needs.

The future for moral debate, to my mind, has to be concerned with the insurmountable dichotomy between complete and consistent moral systems. As we can no longer expect a unique answer to every situation, we have to decide which is more valuable. Do we want to have a system which never gives incompatible advice but leaves areas of our lives unguided? Would you prefer, instead, a moral system which can always answer your quandary even though each of the parts is inconsistent as a whole? Maybe the two systems can live side by side. I have not the answer to this but there is certainly the opportunity for progress to be made.

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VI

Bibliography

Austin, J L	*Ifs and Cans* from *Proceedings of the British Academy* ***XLII***	1956	OUP
Barker, S.F.	*Philosophy of Mathematics*	1964	Prentice-Hall
Benacerraf, P & Hilary, P	*Philosophy of Mathematics, Selected Readings.*	1983 (1997)	CUP
Benn, P	*Ethics*	2005 [1998]	Routledge
Blackburn, S	*Dilemmas: Dithering, Plumping and Grief* from Mason (1996)		
Bradley, F H	selection from: *Ethical Studies* reprinted in Gowans (1987)	1927	

Brandt, R B	*A Theory of the Good and the Right*	1979 (1998)	Prometheus
Brink, D O	*Moral conflict and Its Structure* from *Philosophical Review* ***CIII-ii*** reprinted in Mason (1996)	1994	
Conee, E	*Against Moral Dilemmas* reprinted in Gowans (1987)	1982	
Dahl, N O	*Morality, Moral Dilemmas, and Moral Requirements* from Mason (1996)		
Donagan, A	*Consistency in Rationalist Moral Systems* reprinted in Gowans (1987)	1984	
"	*Moral Dilemmas, Genuine and Spurious: A Comparative Anatomy* from *Ethics* ***CIV-i*** reprinted in Mason (1996)	1993	
Feldman, F	*Introductory Ethics*	1978	Prentice-Hall
Foot, P	*Morality as a System of Hypothetical Imperatives* from *The Philosophical Review* ***LXXXI-iii***	1972	Duke University Press

Foot, P	*Moral Realism and Moral Dilemma*	1983	
	reprinted in Gowans (1987)		
van Frassen, B C	*The Logic of Conditional Obligation* from *Journal of Philosophical Logic* ***I-iv***	1972	D.Reidel
“	*Values and the Heart's Command*	1973	
	reprinted in Gowans (1987)		
Gödel, K	*On Formally Undecidable Propositions of Principia Mathematica and Related Systems* translated by Meltzer, B	1962	Cunningham
Gowans, C W	*Moral Dilemmas* (Ed.)	1987	OUP
“	*Moral Theory, Moral Dilemmas, and Moral Responsibility* from Mason (1996)		
Guttenplan, S	*The Languages of Logic*	1986 [1997]	Blackwell
Hamblin, C L	*Quandaries and the Logic of Rules* from *Journal of Philosophical Logic* ***I-i***	1972	D.Reidel

Hare, R M	selections from:		
	Moral Thinking	1980	
	reprinted in Gowans (1987)		
Hilbert, D	*On The Infinite.*	1925	
	reprinted in Benacerraf (1997)		
Hill, T E Jr.	*Moral Dillemas, Gaps and*		
	Residues: A Kantian Perspective		
	from Mason (1996)		
Honderich, T	*The Oxford Companion to*	1995	OUP
	Philosophy (Ed.)		
Howson, C	*Logic With Trees, An Introduction*	1997	Routledge
	to Symbolic Logic		
Hofstadter, D R	*Gödel, Escher, Bach: an Eternal*	2000	Penguin
	Golden Braid	[1979]	
Kant, I	*Grounding for the Metaphysics of*	1993	Hackett
	Morals translated by Ellington, J W	[1785]	
	selections from:		
“	*The Doctrine of Virtue: Part II of*	1971	
	the Metaphysics of Morals		
	reprinted in Gowans (1987)		
Kierkegaard, S	*Fear and Trembling*, translated by	1985	Penguin
	Hannay, A		

Lemmon, J	*Moral Dilemmas*	1962	
	reprinted in Gowans (1987)		
McConnell, T C	*Moral Residue and Dilemmas*		
	in Mason (1996)		
	Moral Dilemmas and Consistency	1978	
	in Ethics		
	from *Canadian Journal of*		
	Philosophy ***XIII-ii***		
	reprinted in Gowans (1987)		
Mackie, J L.	*Ethics: Inventing Right and Wrong*	1977	Penguin
MacNiven, D	*Creative Morality*	1993	Routledge
Marcus, R B	*Moral Dilemmas and Consistency*	1980	
	reprinted in Gowans (1987)		
	More About Moral Dilemmas		
	from Mason (1996)		
Mason, H E	*Responsibilities and Principles:*		
	Reflections on the Sources of Moral		
	Dilemmas		
	from Mason (1996)		
“	*Moral Dilemmas and Moral*	1996	OUP
	Theory (ed.)		

Mill, J S	*Utilitarianism*	1882	Longmans, Green & Co
	selections from:		
“	*A System of Logic, Ratiocinative and Inductive*	1884	
“	*Utilitarianism*	1961	
	reprinted in Gowans (1987)		
Mothersill, M	*The Moral Dilemmas Debate* from Mason (1996)		
Nagel, E & Newman, J R	*Gödel's Proof*	1958	Routledge
Nagel, T	selection from:		
	Moral Questions	1979	
	reprinted in Gowans (1987)		
Plato	*The Republic* translated by Lee, D	2003 [1955]	Penguin
Railton, P	*The Diversity of Moral Dilemma* from Mason (1996)		
Rawls, J	*A Theory Of Justice*	1971(1973)	OUP
Ross, D	selections from:		
	The Right and the Good	1930	
	reprinted in Gowans (1987)		

Scarre, G	*Utilitarianism*	1996	Routledge
Shanker, S G	*Gödel's Theorem: In Focus*	1988	
Sinnott-Armstrong, W	*Moral Dilemmas and Rights* from Marcus (1996)		
Williams, B	selections from: *Problems of the Self: Philosophical Papers 1956-1972* reprinted in Gowans (1987)	1973	

-§-

VII

Webliography

Edward N. Zalta (ed.),

The Stanford Encyclopedia of Philosophy

http://plato.stanford.edu/

Denis, Lara, "Kant and Hume on Morality",

The Stanford Encyclopedia of Philosophy (Summer 2008 Edition

http://plato.stanford.edu/archives/sum2008/entries/kant-hume-morality/

Johnson, Robert, "Kant's Moral Philosophy",

The Stanford Encyclopedia of Philosophy (Summer 2008 Edition),

http://plato.stanford.edu/archives/sum2008/entries/kant-moral/

Garson, James, "Modal Logic",

The Stanford Encyclopedia of Philosophy (Spring 2008 Edition),

http://plato.stanford.edu/archives/spr2008/entries/logic-modal/

Hsieh, Nien-hê, "Incommensurable Values",

The Stanford Encyclopedia of Philosophy (Fall 2007 Edition),

http://plato.stanford.edu/archives/fall2007/entries/value-incommensurable/

McConnell, Terrance, "Moral Dilemmas",

The Stanford Encyclopedia of Philosophy (Summer 2006 Edition),

http://plato.stanford.edu/archives/sum2006/entries/moral-dilemmas/

McNamara, Paul, "Deontic Logic",

The Stanford Encyclopedia of Philosophy (Spring 2006 Edition),

http://plato.stanford.edu/archives/spr2006/entries/logic-deontic/

Rauscher, Frederick, "Kant's Social and Political Philosophy",

The Stanford Encyclopedia of Philosophy (Fall 2007 Edition),

http://plato.stanford.edu/archives/fall2007/entries/kant-social-political/

Richardson, Henry S., "Moral Reasoning",

The Stanford Encyclopedia of Philosophy (Fall 2007 Edition),

http://plato.stanford.edu/archives/fall2007/entries/reasoning-moral/

-§-